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OPTIMAL WIRE SIZE FOR PHOTOVOLTAIC SYSTEMS OPERATING AT MAXIMUM POWER POINT: A CLOSED FORM APPROACH

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ABSTRACT

While the purchase of cable represents an admittedly minor part of the cost of installing a photovoltaic system, it is a cost nonetheless, and one that should be examined with rigor by a maturing photovoltaic industry. Using larger cables between array and point of connection to the battery or grid-tied inverter reduces array power losses, but increases the costs of the cables. In the past, little attention has been paid to the optimization of the sizing of the cables; the effort of doing a simulation study did not appear to justify the potential gains.

This study shows that, in fact, a very simple relation can be used to determine the financially optimum cable size. This simple relation is derived from basic considerations of the costs associated with wiring losses and the wiring itself. The challenge is to determine average level of losses, since these will depend on the varying level of sunshine over the course of a year. This study uses a simple exponential expression for the frequency distribution of array power output; this distribution is characterized by three parameters: the number of daylight hours in the year, the annual output of the array, and the maximum power output of the array. This simple distribution has the advantage of giving rise to an integrable expression for the frequency distribution of power losses in the cables. The result can be incorporated into an expression for the total cost of the cables (costs associated with purchase and losses), which can then be minimized, yielding a simple expression for the optimum cable size.

This closed form solution demonstrates that the length of the cable run is irrelevant to the optimal cable size. This somewhat counterintuitive result is explained in the development of the closed form solution itself.

The predictions of this closed form solution are compared to simulations to establish its validity. The application of the close-form relation to the problem of cable selection is then illustrated with an example. The method is also used to comment on the Canadian Electrical Code stipulations for wiring.

NOMENCLATURE

C	Constant in the assumed frequency distribution of array output
c_{loss}	The present value of the energy dissipated as heat in the cabling
c_{pv}	The per unit cost of photovoltaic array capacity
E_{array}	Annual energy output of the array
f_{site}	Cabling loss factor, the typical power losses in the cable as a fraction of losses occurring at STC
i	Current in the cable connecting the array to the power converter
$\bar{i}_{squared}$	Average squared current output of the array
l_{wire}	Length of the cable connecting the array to the power converter
$\hat{P}$	Nominal array power at STC

P_{array} Power output of the array at a given point in time
p_{array} Power output of the array at a given time, normalized with respect to nominal array power
$\overline{p}_{array}$ Average power output of the array, normalized with respect to nominal array power
$\hat{P}_{loss}$ Power losses in the cable when the array is operating at STC
$\overline{P}_{loss}$ Average power loss in the cables
P_{loss} Instantaneous power dissipated in the cable connecting the array to the power converter
p_{max} Maximum power output of the array, normalized with respect to nominal array power
R_{wire} Total resistance of cable connecting the array to the power converter
r_{wire} Per unit length resistance of the cable connecting the array to the power converter
$\hat{r}_{wire}$ Optimal per unit length resistance of the cable connecting the array to the power converter
STC Standard test conditions for photovoltaic modules
t_{day} Total time during year when array furnishes power, i.e., total time of daylight in year
V_{array} Array voltage
γ Constant in the assumed frequency distribution of array output

INTRODUCTION

Cabling for a photovoltaic system represents a small, but not insignificant, part of the cost of a photovoltaic system; its relative importance tends to increase with decreasing nominal array voltage and when the array and the point of interconnection are distant from one another. Cable size is normally selected by rules of thumb related to the voltage drop in the cables. The Canadian electrical code stipulates that, in relation to the system voltage, the voltage drop in the cables must not exceed 5% between the array and the point of interconnection and 3% between any two junction boxes in that interconnection (Canadian Standards Association, 2002a); an installer is free to reduce this voltage drop to, say, 1%, if losses are to be minimized.

The choice of the optimal cable size has received little attention. As a consequence of this lack of analysis, a number of questions frequently arise concerning choice of cable, such as:

- What is the monetary benefit of shifting to a higher array voltage, e.g., 48 V instead of 12 V?
- If the array is very far from the point of interconnection, should the voltage drop be minimized, e.g., 1%, or permitted to rise to the maximum allowable, 5%?
- What is the optimal cable size for a given installation?

These questions can be answered relatively easily using a closed form solution for choosing the optimal cable size. While the derivation of this solution is somewhat involved, the ultimate result is very simple.

POWER LOSSES IN CABLING

Power Losses at a Given Point in Time: The power dissipated in the cable connecting the array to the power converter (i.e., inverter or battery charger with maximum power point tracking), P_{loss}, can be found from:

$$P_{loss} = i^2 R_{wire} \qquad \text{Eq. 1}$$

Since power is the product of current and voltage, this leads to

$$P_{loss} = \frac{P_{array}^2 R_{wire}}{V_{array}^2} \qquad \text{Eq. 2}$$

Here V_{array} is assumed constant; the validity of this assumption will be established later. In the meantime, it is sufficient to consider that operation at an appropriately chosen fixed voltage results in losses of only 2 to 4% compared with maximum power point tracking (Jantsch et al., 1992), (Freilich and Gordon, 1991), so operationally the two are not that different. Equation 2 can be recast using a normalized form for the array output:

$$P_{loss} = \frac{\left(\hat{P} \cdot p_{array}\right)^2 R_{wire}}{V_{array}^2} \qquad \text{Eq. 3}$$

where $\hat{P}$ is the nominal array power at STC, such that:

$$p_{array} = \frac{P_{array}}{\hat{P}}. \qquad \text{Eq. 4}$$

Typical Power Losses in the Cable: While Equation 3 can be used to give the power losses in the cable at a given power output, it gives no information about the power levels that will be typical and that should be used when calculating optimal cable sizes. That is, the power will vary continuously with the irradiance on the array and the array temperature, and the frequency distribution of irradiance must be used to determine an average power loss.

A simple approach for estimating the frequency distribution of array output is presented by Peippo and Lund (1994) based on (Bendt et al., 1981). In this approach, the frequency distribution is assumed to be:

$$f\left(p_{array}\right) = Ce^{\gamma p_{array}} \qquad \text{Eq. 5}$$

The constants C and γ are found from solving the following equations, expressed in terms of the normalized maximum and average power output of the array:

$$\bar{p}_{array} = \frac{\left(p_{\max} - \frac{1}{\gamma}\right)e^{\gamma p_{\max}} + \frac{1}{\gamma}}{e^{\gamma p_{\max}} - 1} \qquad \text{Eqs. 6 \& 7}$$

$$C = \frac{\gamma}{e^{\gamma p_{\max}} - 1}$$

The normalized average power output of the array can be found from the annual output of the array and the time during a year when the array furnishes power, i.e., total daylight time:

$$\bar{p}_{array} = \frac{E_{array}}{\hat{P}t_{day}} \ . \qquad \text{Eq. 8}$$

Combining the frequency distribution and the relation for power loss, the typical power loss can be found from integration:

$$\overline{P}_{loss} = \int_0^{p_{\max}} \frac{\left(\hat{P}\cdot p_{array}\right)^2 R_{wire} f\left(p_{array}\right)}{V_{array}^2} dp_{array}$$

$$= \frac{CR_{wire}\hat{P}^2}{V_{array}^2} \int_0^{p_{\max}} p_{array}^2 e^{\gamma p_{array}} dp_{array}$$

$$= \frac{CR_{wire}\hat{P}^2}{V_{array}^2} \left[\frac{e^{\gamma p_{\max}}}{\gamma} \left(p_{\max}^2 - \frac{2p_{\max}}{\gamma} + \frac{2}{\gamma^2} \right) - \frac{2}{\gamma^3} \right] \qquad \text{Eq. 9}$$

$$= \frac{f_{site} R_{wire} \hat{P}^2}{V_{array}^2} \quad where\ f_{site} = C\left[\frac{e^{\gamma p_{\max}}}{\gamma} \left(p_{\max}^2 - \frac{2p_{\max}}{\gamma} + \frac{2}{\gamma^2} \right) - \frac{2}{\gamma^3} \right]$$

Note that the power losses in the cable when it is operating at STC are given by

$$\hat{P}_{loss} = \frac{R_{wire}\hat{P}^2}{V_{array}^2} \qquad \text{Eq. 10}$$

and therefore

$$\overline{P}_{loss} = f_{site}\hat{P}_{loss}\ . \qquad \text{Eq. 11}$$

Thus, f_{site}, the cabling loss factor, indicates the magnitude of average cabling power losses as a fraction of the losses that would occur if the array always put out its rated power. This is interesting and useful. For example, consider a site where the annual output of the array is 1000 kWh/kWp, the output of the array does not exceed its nominal power, and there are 4000 daylight hours. Then the normalized average power output of the array, $\overline{p}_{array}$, is 0.25. From this, the constants C=3.7 and γ=-3.6 can be calculated and f_{site} is found to be 0.11: the average losses in the cabling are only 11% of those occurring when the array is operating at STC. Therefore, the rules normally used to size the cabling essentially assume cabling power losses an order of magnitude larger than they are, on average, for this installation!

MINIMIZATION OF TOTAL WIRING COSTS

Formulation: The objective of optimizing the wiring is to select R_{wire} such that total wiring costs are minimized. Total wiring costs consist of 1) the installed cost of the cables as a function of the resistance, $c_{wire} = c_{wire}(R_{wire})$, and 2) the cost of the energy dissipated as heat in the wires.

Fortunately, the cost of the energy dissipated as heat in the wires can be expressed in terms of the cost of photovoltaic capacity:

$$c_{loss} = \frac{\overline{p}_{loss}}{\overline{p}_{array}} \cdot \hat{P} \cdot c_{pv} \qquad \text{Eq. 12}$$

In essence, for the present value of the stream of future energy benefits forfeited as heat in the wires we are substituting the present value of the additional array capacity that would generate this same stream of future benefits. That this is the case can be shown by considering an array that on average furnishes x amount of power to the converter. Then

$$x = \hat{P} \cdot \left(\overline{p}_{array} - \overline{p}_{loss}\right) \qquad \text{Eq. 13}$$

so

$$\hat{P} = \frac{x}{\overline{p}_{array} - \overline{p}_{loss}} \qquad \text{Eq. 14}$$

We can then determine how much the array must be enlarged due to wiring losses, by calculating the difference in array size between an array with losses and an array without losses:

$$\begin{aligned}\Delta\hat{P} &= \hat{P}_{with\ cable\ losses} - \hat{P}_{without\ cable\ losses} \\ &= \frac{x}{\overline{p}_{array} - \overline{p}_{loss}} - \frac{x}{\overline{p}_{array}} \\ &= \frac{\overline{p}_{loss}}{\overline{p}_{array}} \cdot \hat{P}\end{aligned} \qquad \text{Eq. 15}$$

Multiplying this by the per unit cost of photovoltaic capacity yields c_{loss}, as in Eq. 12.

Thus total wiring costs are given by

$$\begin{aligned}c_{tot} &= c_{loss} + c_{wire} \\ &= \frac{\overline{p}_{loss}}{\overline{p}_{array}} \hat{P} \cdot c_{pv} + c_{wire}\left(R_{wire}\right) \\ &= \frac{f_{site} R_{wire} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + c_{wire}\left(R_{wire}\right)\end{aligned} \qquad \text{Eq. 16}$$

This can be expressed in terms of the per unit length resistance of the wire, r_{wire}:

$$c_{tot} = \frac{f_{site} l_{wire} r_{wire} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + c_{wire}\left(r_{wire}\right) \cdot l_{wire} \qquad \text{Eq. 17}$$

The minimum total wiring costs will be found by setting to zero the derivative of this expression with respect to r_{wire}:

$$\frac{dc_{tot}}{dr_{wire}} = \frac{f_{site} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + \frac{dc_{wire}}{dr_{wire}} = 0 \qquad \text{Eq. 18}$$

Note that dividing by l_{wire} has eliminated it from Equation 18.

Cable Costs: In general, it can be expected that the per unit cost of cabling will have a fixed portion and a portion inversely related to the resistance of the cable (this being proportional to the amount of copper and insulation used in the cable). Thus,

$$c_{wire}\left(r_{wire}\right) = c_{wfix} + \frac{c_{wvar}}{r_{wire}} \qquad \text{Eq. 19}$$

For example, plotting cost (according to one Canadian supplier) versus per unit length resistance for RW90 stranded copper cable yields Figure 1. The data cover 14 AWG up to 750 000 circular mils—a vast range, as suggested by the log scale on the figure. The line fit to these data with c_{wfix} = 0.09 \$/m and c_{wvar} = 0.00182 \$·ohm/m is nearly perfectly coincident with the data; errors, also shown on the figure, are generally less than 6%. It would be possible to adjust these costs to include the cost of conduit and installation.

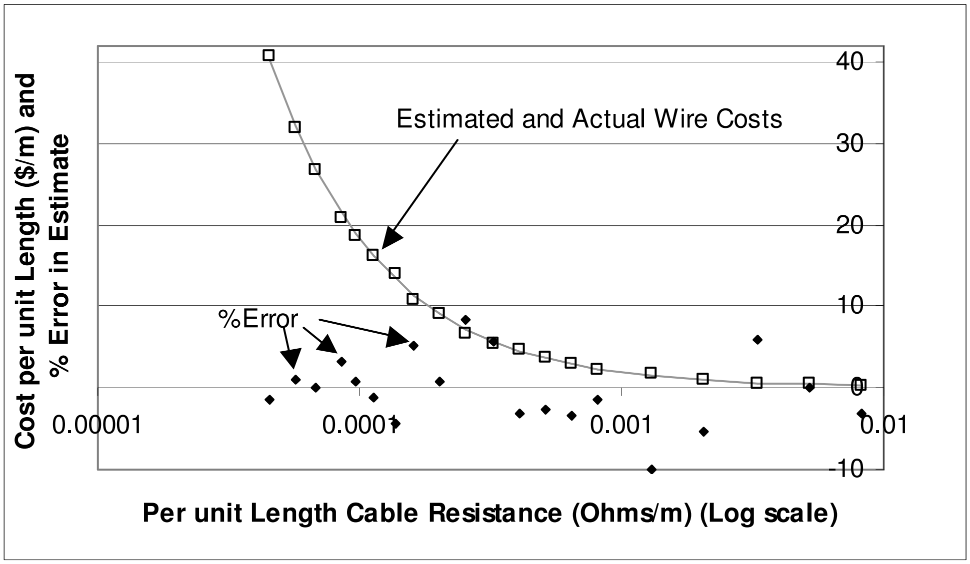

Figure 1: Cost versus Resistance: Estimated and Actual for RW90 Stranded Copper Cable

Optimal Cable Resistance: Combining Equations 18 and 19 leads to an equation for the optimal resistance of the wire, $\hat{r}_{wire}$:

$$\frac{f_{site}\hat{P}^2}{V_{array}^2\,\overline{p}_{array}}c_{pv} - \frac{c_{wvar}}{\hat{r}_{wire}^2} = 0$$

$$\hat{r}_{wire} = \frac{V_{array}}{\hat{P}}\sqrt{\frac{\overline{p}_{array}c_{wvar}}{f_{site}c_{pv}}} \qquad \text{Eq. 20}$$

For example, at the installation mentioned earlier, producing 1000 Wh/Wp and with cabling loss factor of 0.11, wiring costs as in Figure 1, and a PV capacity cost of $10/Wp, the optimal per unit wire resistance is given by the following equation, which is no more difficult to use than, for example, a 5% voltage drop rule.

$$\hat{r}_{wire} = \frac{V_{array}}{49\hat{P}} \qquad \text{Eq. 21}$$

It should be noted that there is no reference to the length of the conductor in this relation. This demonstrates that cable length is not a consideration in the choice of optimum size of conductor. Furthermore, the elimination of l_{wire} from Equation 18 is not contingent upon the validity of the assumed distribution for array output: even if f_{site} were incorrectly estimated, it would still be possible to eliminate l_{wire} from the equation.

To some the absence of the length of the cable run may seem counterintuitive or surprising. It is helpful to think about this in terms of a unit length of cable: this will have a particular purchase cost associated with it and it will cause a certain energy loss, with an associated cost. The optimal wiring resistance balances these two costs. A longer length of cable multiplies each of these costs by the same factor, so that total costs are higher, but the balance between the two is unchanged.

COMPARISON WITH SIMULATION

The validity of the above approach—in particular, the use of the exponential frequency distribution and the assumption that the array voltage is constant—was verified through simulations with the PVToolbox simulation package (Sheriff et al., 2003). Hourly weather data were used to determine the output of a single photovoltaic module (see Table 1 for specifications), operating at its maximum power point. For each simulated year, the integral of the squared array current was divided by the daylight hours to find the average squared current for the year. This was, in turn, averaged over all the years of simulation for the site and array orientation. Note that the energy dissipated as heat in the wires is equal to the product of the wire resistance (assumed constant) and the squared current. The average maximum power point voltage, weighted by the power output of the array rather than time, was also determined in the simulation. Then, rearranging Equation 9, the cabling loss factor, f_{site}, implied by the simulation was calculated from:

$$f_{site} = \frac{\bar{i}_{squared} \cdot V_{array}^2}{\hat{P}^2} \qquad \text{Eq. 22}$$

The average squared current is written in this peculiar way to emphasize that it is not merely the square of the average current.

Parameter	Value
Rated voltage	17.4 V
Rated current	3.16 A
Open-circuit voltage	21.7 V
Short-circuit current	3.40 A
Nominal Operating Cell Temperature (NOCT)	45 ºC
Open circuit voltage temperature coefficient	-0.350 %/ºC
Short-circuit current temperature coefficient	0.041 %/ºC
Efficiency (for cell temperature estimate only)	12 %

Table 1: PV Module Characteristics

The CWEEDS data set (Environment Canada, 2003) provided hourly measurements of ambient air temperature and the insolation on the horizontal for sites within Canada. Only sites having measured, rather than modeled, values for the horizontal insolation were chosen, and only years with a large majority of their data being measured values were used. Thus, for each of five sites (Vancouver, Inuvik, Edmonton, Toronto, and St. John's), the simulation ran through between 20 and 46 years of weather data.

For locations outside of Canada, hourly data were not available. Rather, average monthly values for temperature and clearness index (Duffie and Beckman, 1991) served as inputs to the Watgen synthetic weather generator (Watsun, 1992). As a check of the synthesized weather data, the value of the cabling loss factor calculated using the synthetic data was compared to that calculated using CWEEDS data for five array orientations in Toronto and one orientation in each of Edmonton and Inuvik; the agreement was within several percent.

Table 2 compares the values of f_{site} from simulation and the above-described analytical method. The average relative error is for these sites is –7.1%; errors tend to be largest for east and west facing arrays. This level of accuracy should be acceptable for the purposes of wiring optimization. It should be noted that in applying the analytical method, the values of average maximum power point voltage, average array output, and maximum array output were taken from the simulation.

APPLICATIONS

Finding Optimal Wire Size for an Installation: Consider an example taken from the web site of a charge controller manufacturer. A nominally 360 W, 12 V array is located 60 m from the point of interconnection (i.e., a total wiring distance of 120 m). If the wire size is selected to achieve a 5% voltage drop, then the wire resistance should be $1.67 \cdot 10^{-4}$ Ω/m. The closest wire size is 4/0, with a resistance of $1.61 \cdot 10^{-4}$ Ω/m. This wire costs $10.86/m according to one supplier, and thus wiring costs are $1303.

Site	Data	Azimuth	Tilt	V_{mpp}	Output	f_{site}		Error
	*	º	º	V	Wh/Wp	Sim.	Analytic	%
Vancouver	C	0	50	16.34	1285	0.179	0.158	-11.9%
Vancouver	C	0	0	16.17	1090	0.121	0.112	-7.3%
Vancouver	C	0	90	16.27	947	0.103	0.094	-9.2%
Vancouver	C	90 (W)	90	15.98	749	0.070	0.060	-13.8%
Vancouver	C	-90 (E)	45	16.16	919	0.101	0.086	-14.9%
Inuvik	C	0	70	17.28	1149	0.183	0.156	-14.7%
Inuvik	W	0	70	16.63	1110	0.170	0.144	-15.4%
Inuvik	C	0	0	16.68	822	0.078	0.074	-4.6%
Inuvik	C	0	90	17.31	1002	0.145	0.124	-14.3%
Inuvik	C	0	30	17.09	1113	0.156	0.139	-10.8%
Inuvik	C	90 (W)	90	16.72	670	0.071	0.058	-18.9%
Inuvik	C	-90 (E)	45	16.87	785	0.089	0.074	-16.2%
Edmonton	C	0	55	16.82	1565	0.230	0.225	-2.1%
Edmonton	W	0	55	16.89	1637	0.256	0.244	-4.6%
Edmonton	C	0	0	16.39	1147	0.120	0.119	-0.6%
Edmonton	C	0	90	16.89	1278	0.163	0.161	-1.3%
Edmonton	C	90 (W)	90	16.3	810	0.075	0.069	-7.4%
Edmonton	C	-90 (E)	45	16.54	1097	0.127	0.116	-9.1%
Toronto	C	0	45	16.47	1381	0.192	0.183	-4.6%
Toronto	W	0	45	16.42	1360	0.190	0.180	-5.5%
Toronto	C	0	0	16.24	1192	0.136	0.132	-3.3%
Toronto	W	0	0	16.23	1183	0.136	0.134	-1.3%
Toronto	C	0	90	16.5	995	0.108	0.104	-3.7%
Toronto	W	0	90	16.42	984	0.108	0.104	-4.4%
Toronto	C	90 (W)	90	16.03	737	0.061	0.059	-4.1%
Toronto	W	90 (W)	90	16	767	0.070	0.064	-8.6%
Toronto	C	-90 (E)	45	16.3	1083	0.126	0.114	-9.4%
Toronto	W	-90 (E)	45	16.29	1080	0.129	0.117	-9.4%
Toronto	C	180 (N)	45	15.66	629	0.036	0.036	-1.1%
St. John's	C	0	50	16.7	1194	0.158	0.145	-8.6%
St. John's	C	0	0	16.48	1017	0.106	0.102	-3.8%
St. John's	C	0	90	16.68	901	0.097	0.089	-8.5%
St. John's	C	90 (W)	90	16.29	684	0.058	0.052	-10.4%
St. John's	C	-90 (E)	45	16.49	895	0.094	0.083	-10.9%
Albuquerque, NM	W	0	35	16.24	2193	0.354	0.349	-1.3%
Albuquerque, NM	W	0	0	16.08	1896	0.265	0.269	1.3%
Albuquerque, NM	W	0	90	16.29	1402	0.165	0.170	2.8%
Albuquerque, NM	W	90 (W)	90	15.79	1122	0.128	0.117	-8.2%
Albuquerque, NM	W	-90 (E)	45	16.23	1665	0.241	0.223	-7.6%
Athens, Greece	W	0	40	15.88	1524	0.210	0.204	-2.9%
El Fasher, Sudan	W	0	15	15.55	2133	0.323	0.324	0.3%
* C indicates CWEEDS data, W indicates typical year synthesized by Watgen								

Table 2: f_{site} as Calculated by PVToolbox Simulation and Proposed Method

Let us now apply the analytical method, and forget for a moment that the Canadian Electrical Code limits the voltage drop to 5%. We assume that this array will be installed near Vancouver, facing due south, at a tilt angle of 50º, and will cost $10/Wp. To get a quick estimate of the average output of the array, we use the RETScreen PV model (RETScreen, 2005), with no losses in inverter or array: it suggests 1281 Wh/Wp. We assume that there are 4000 daylight hours in the year, that the array will at maximum produce 360 W, and that its average maximum power point voltage will be 16.5 V. On this basis, we calculate the normalized average power output of the array, $\overline{p}_{array}$, to be 0.321 and the cabling loss factor, f_{site}, to be 0.167 (compare with a simulated value of 0.179, in Table 2). Then, recalling Equation 20 and the cost information of Figure 1, we find the optimal per unit length wire resistance:

$$\begin{aligned}\hat{r}_{wire} &= \frac{V_{array}}{\hat{P}}\sqrt{\frac{\overline{p}_{array}c_{wvar}}{f_{site}c_{pv}}} \\ &= \frac{16.5}{360}\sqrt{\frac{0.321\cdot 0.00182}{0.167\cdot 10}} \\ &= 8.6\cdot 10^{-4}\ \Omega/\mathrm{m}\end{aligned}$$

The closest wire size is AWG #4, which has a resistance of $8.2\cdot 10^{-4}$ Ω/m. This wire is half as large in diameter as the 4/0 wire suggested by the 5% voltage drop rule, and costs $2.36/m, rather than $10.86/m. Thus wiring costs fall from $1303 to $283—a difference of roughly 20% of the total initial cost of this system! The total wiring costs, that is, with the cost of losses included, are found from Equation 17. With the 4/0 wire, they are $1351, and with the #4 wire, they are $526—again, a very substantial difference.

This is illustrated in Figure 2, which shows the wiring purchase cost, the cost of losses and the total wiring cost. With the 4/0 wire, wiring purchase costs constitute virtually the entirety of the total wiring costs, with the cost of losses only $48—the equivalent of 5 Wp of additional array capacity. At the optimal wire size of AWG #4, the power losses are worth $243, nearly equal to the purchase cost.

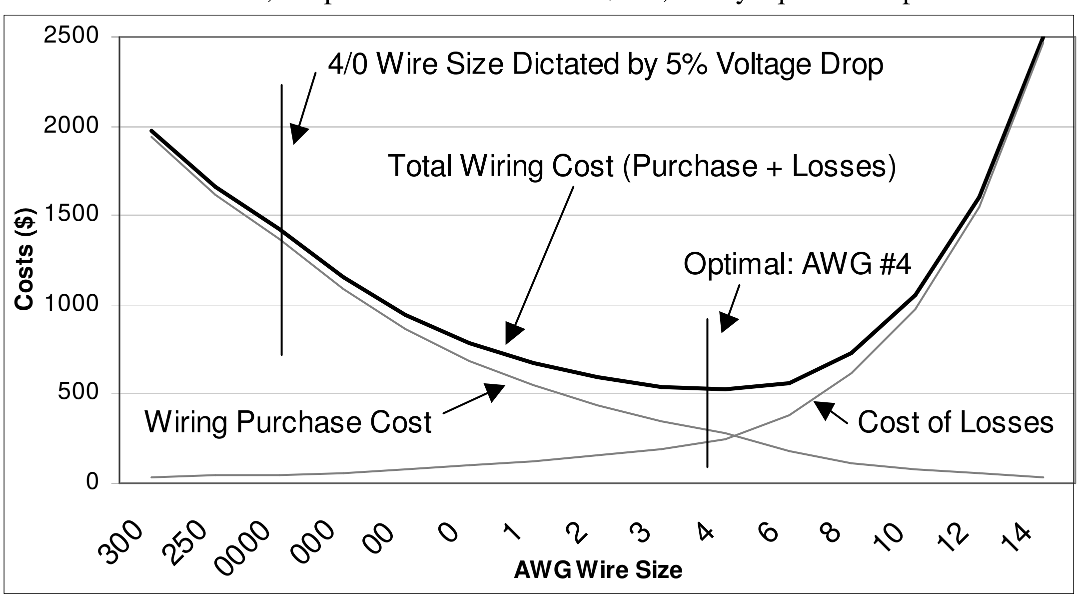

Figure 2: Finding the Optimal Wiring Size for a Nominally 360 W, 12 V Array with 120 m of Wiring

This example is probably atypical in that the financial penalty of following the 5% voltage drop rule is so large. This largely stems from it being a 12 V system, with high array currents in comparison to the size of the system. But the 5% voltage drop rule can lead the system designer astray, even at higher voltages, and not always by specifying a suboptimally large wire. For example, if the above system was actually a nominally 1000 W, 48 V array with a 15 m total wiring distance, then the 5% voltage drop rule would permit an AWG #14 wire to be used. Reusing the calculations from the previous example, for this site and array orientation:

$$\hat{r}_{wire} = \frac{V_{array}}{53.5 \cdot \hat{P}}$$

The optimal wire resistance is $1.2 \cdot 10^{-3}$ Ω/m, which is most closely approximated by AWG #6. On the basis of this simple calculation, the designer would pay $18 more in the purchase cost of the cabling but save $125 in reduced losses.

Note also that the optimal wiring size lies in a broad valley. The small errors in the analytical method presented here do not, therefore, much affect the results. Using the simulated value for f_{site} from Table 2 in the above two examples, AWG #4 and #6 would have still been chosen.

Benefit of Switching to a Higher Voltage: The first example in the previous section came from the web site of a controller manufacturer. They used this example to show that a controller that permitted the array to operate at a voltage much higher than the system voltage would pay for itself in the reduced wiring costs. They argued that with the same 360 W array wired in a 48 V configuration, wiring savings of around $1000 would result, and their controller would still permit a 12 V system voltage.

What is the real value of such a controller? The expression for the optimal wire resistance given above indicates that with a nominally 48 V array, the optimal wire resistance is $3.4 \cdot 10^{-3}$ Ω/m. AWG #10 would be used (by chance, it would also have been the choice using the 5% rule). The wiring purchase price would fall by around $200 compared with the #4 that is optimal for the 12 V array. Total wiring costs would be around $140, or a reduction of a little under $400. This demonstrates how the approach developed here can facilitate the investigation of this type of problem.

Implications for the Canadian Electrical Code: Unfortunately, the Canadian Electrical Code (Canadian Standards Association, 2002a) renders moot much of the above discussion. Rule 8-102 dictates that the voltage drop shall not exceed 5% from the supply side to the point of utilization, and not exceed 3% in a feeder or branch circuit.

The rationale for this rule is that the voltage drop "may result in a lower than acceptable application voltage…[that] will decrease the operating efficiency of electrical equipment such as motors, heating systems, and lighting systems. The establishment of criteria for maximum allowable voltage drop in a circuit ensures acceptable utilization voltages, to obtain optimum performance from electrical equipment" (Canadian Standards Association, 2002b). This rationale does not apply to photovoltaic systems with maximum power point tracking: over the course of a day, natural variation in the irradiance will cause the maximum power point voltage to vary over a large range, bounded on the low side by zero. A higher voltage drop will lead to more rapid changes in the maximum power point voltage when irradiance conditions change, but it seems perverse to limit the natural variation of the system when there is no good technical reason that the system should not be able to accommodate it. This is especially true when the rule can dictate significant additional costs that do not add to system safety.

A fundamental flaw in the rule is that it relates the choice of the wire to its length. This paper has shown that the optimal wire size is independent of the wire length. The use of the rule will, therefore, necessarily lead to suboptimal wire size choices in certain systems. Based on these arguments, the Canadian Electrical Code should except photovoltaic systems with maximum power point trackers from Rule 8-102.

CONCLUSIONS

A simple exponential function can be used to approximate the frequency distribution of the output of a photovoltaic array connected to a maximum power point tracker. Because it is easily integrated, it can be used to express, in closed form, the annual average losses in the wiring connecting the array and the point of interconnection. With wire purchase costs expressed, with good accuracy, as a simple function of the wiring resistance, the total wiring costs, comprising losses and wiring purchase costs, can be minimized by setting the derivative to zero and solving for the resistance.

A unit length of wire between the array and the point of interconnection incurs certain losses due to its resistance. The present value of this stream of losses can be found by determining the purchase cost of the array capacity that would generate energy equivalent to these losses. The optimal wiring size can be found by considering a unit length of wire: that is, the length of the wiring run is irrelevant.

Presently, the Canadian Electrical Code stipulates that the voltage drop in the wiring between the array and the point of interconnection shall not exceed 5%, or, depending on the installation, 3% of the system voltage. Since this relates the choice of wire size directly to the length of the wiring run, it will necessarily lead to suboptimal choice of wiring sizes in many installations. In certain cases, the difference between the wiring costs of the optimally sized conductor and the conductor chosen on the basis of the 5% rule can be in the neighborhood of 20% of the total costs of the PV system. Since the rationale for the Electrical Code rule does not make sense in the context of photovoltaic systems, this article argues that photovoltaic systems with maximum power point trackers should be excepted from Rule 8-102 of the Canadian Electrical Code.

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