A SIMPLE, RIGOROUS METHOD FOR SIZING THE ARRAY OF A PV HYBRID SYSTEM

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ABSTRACT

Many rules of thumb for sizing the array for a PV-hybrid system have been developed. The rules of thumb include sizing the array so that it meets the average load under the annual average solar irradiance, so that it meets (say) 60% of the annual load, so that it fully satisfies the load during half the year, or so that no more than (say) 25% of its output is rejected in the sunniest month of the year. All of these result in reasonable array sizes under certain conditions. But all of them ignore the financial aspects of array sizing, so in other conditions they may yield results that will not be cost-effective. Ultimately, the optimal array size is based on financial considerations: it minimizes the cost of providing power.

Sizing and simulation software have also been developed for this purpose. Unfortunately, they rarely reveal why a certain sizing is appropriate, so the factors influencing the sizing are not clear. This makes them prone to user error. If simulation software does not automatically search for the best array sizing, it is up to the user to try multiple sizings and figure out the best one. Furthermore, sometimes it is inconvenient to have to learn a piece of software and use a computer.

In fact, the optimal sizing of the PV array does not require simulation, nor is a rigorous solution so complicated as to necessitate rules of thumb. Here a sizing method is presented that is very simple, that makes sense intuitively, that finds the most cost-effective array size, and that can be applied with a pen, paper, and monthly solar data in tabular form—or adapted to the most sophisticated simulation software.

THE OPTIMISATION PROBLEM

Understanding the optimization problem behind array sizing leads directly to the method described here [Smiley et al., 2000], [Ross, 2005]. There are only two costs of any major significance to this problem: the installed cost of the array and the present value of all future outlays for genset operation (which includes fuel, maintenance, overhaul, and replacement expenses). Costs associated with the battery, while accounting for a major chunk of the lifecycle costs of the system, barely influence the optimal array size as long as the battery has sufficient capacity to operate the load for around two days or more [Celik, 2002]. Therefore, they will be largely ignored here.

Consider a genset-battery system. Add a photovoltaic array, module by module. The cost of the system evolves very predictably. Initially, when there is no PV array, costs are entirely due to genset fuel and maintenance outlays. When the first PV module is added, the installed cost of the array appears. But it generates some electricity, such that the genset runs less. Thus the costs associated with the genset fall.

With every module that is added, the cost of the array rises; here it is assumed that it rises linearly with the size of the array. With every module added, the cost associated with the genset falls. Initially this cost falls linearly, too[1]. But at some point, the array will be so large that at times its output will be rejected by the charge controller—the PV array current will be larger than the load and the current that the battery is able to accept (due to being at a high state-of-charge). The cost associated with genset operation

[1] This is an approximation that is acceptable for the purposes of this method. In reality, fuel costs will not fall in a straight line because initially all PV output can be used directly by the load, but eventually some of it will have to be stored in the battery, undergoing certain losses, and thus reducing genset operation less.

will no longer fall so quickly; eventually it will not fall at all, since the load has either been met year round by PV or there are periods of so little sunlight that PV does not function (e.g., polar night).

This is shown in Figure 1, below. The rising line for array cost and the falling line for genset costs are combined to form a third line, overall cost. This has a low point, where system costs are at a minimum. The array size at this point corresponds to the optimal array size. But note also that if a somewhat smaller or larger array is chosen, the increase in costs is insignificant—increases in one cost are offset by decreases in the other. Thus, there is latitude to alter the array sizing, and it should be acceptable to make some approximations in the analysis.

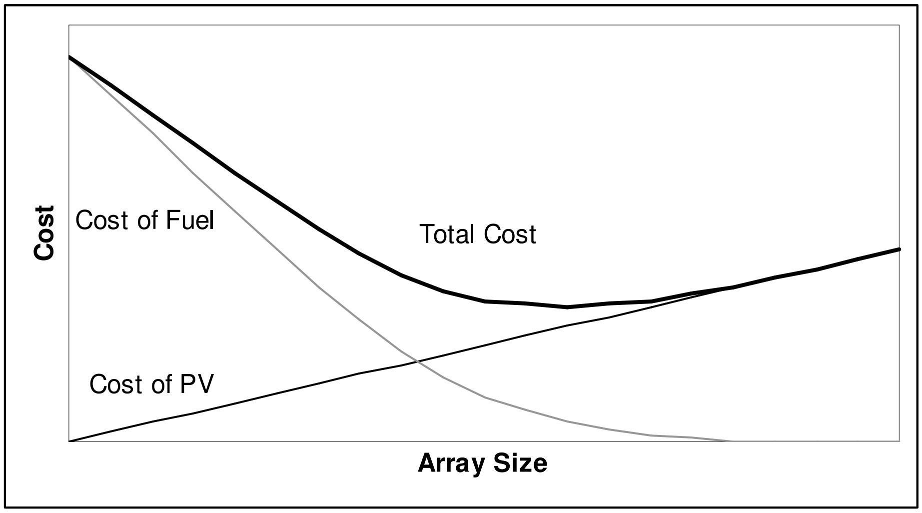

Figure 1: Graphical Representation of the Optimisation Problem (from [Smiley et al., 2000])

In Figure 1, when the first units of array capacity are added, they generate electricity more cheaply, on a per unit basis, than does the genset. Therefore, it makes sense to use more array capacity. Once the added array capacity makes the array so large that some of its output is rejected by the charge controller, the per unit cost of generating electricity with PV rises. The point of minimum cost occurs where the per unit cost of electricity generated by an additional unit of PV array capacity is equal to the cost of generating it with the genset (this cost is assumed to be fixed regardless of the size of the array). It does not make sense to add more PV array capacity beyond this point, since its output could be generated more cheaply with the genset.

Thus, the optimization problem boils down to the following [Ross and Turcotte, 2007]:

1) Calculating C_{PV}, the per unit cost of generating electricity with the PV array assuming that none of it is wasted;
2) Calculating C_{genset}, the per unit cost of generating electricity with the genset.
3) Sizing the array such that were another unit of PV array capacity added, the fraction of the additional unit's output that would be wasted over the course of the year, $f_{PVWaste}$, would be:

$$f_{PVWaste} = 1 - \frac{C_{PV}}{C_{genset}}$$

DETERMINING THE COST OF GENERATING ELECTRICITY WITH PV

The per unit cost of generating electricity with the PV array assuming none of it is wasted, C_{PV}, is calculated by:

$$C_{PV} = \frac{c_{array\ capacity}}{E_{array}}$$

where $c_{array\ capacity}$ is the installed cost per unit of array capacity, and E_{array} is the energy produced and provided to the load or the battery (assuming nothing is rejected by the charge controller) by each unit of array capacity over the entire life of the project.

In Canada, $c_{array\ capacity}$ is generally around \$8/W_p for accessible sites. If necessary, the estimate can be adjusted to account for financing costs, such as debt service payments: calculate the present value of the future stream of debt payments, add it to the equity portion of the installed cost of the array, and then divide by the size of the array[2].

The useful energy produced by each unit of array capacity over the project lifetime, E_{array}, is found quite simply: multiply the project duration, in years, by the annual output of a unit of array capacity, reduced by losses in the battery, wiring, and any power conversion equipment, *but assuming no electricity is rejected by the charge controller*.

This can be done quite approximately. As an example, consider an installation at Winnipeg, Manitoba, and account for all losses and reductions step by step:

- A south facing fixed array tilted at the latitude (50º) will receive around 1670 kWh/m^2 of solar energy per year. If the array was kept clean throughout the year, the sun shone brightly and directly onto the array whenever it did shine, and the array was always operated at the voltage that maximized its output (its maximum power point), one would expect a 1 W_p array to generate 1.67 kWh in a year (since the annual output of a 1 W_p array, in kWh, will be roughly equal to the annual solar energy impinging on a identically oriented one square metre surface, expressed in MWh).
- The array will not always be free of dust, water, or snow, so reduce the output by, say, 3% (to 1.62 kWh/W_p)[3].
- No correction for temperature is necessary for this example: in the summer, the array will be hot, reducing the power available to the maximum power point tracker, but in winter in Winnipeg it will cold, and this will compensate. At warmer sites in Canada a reduction on the order of a few percent may be warranted.
- If a charge controller with maximum power point tracking is the interface between the array and the battery, reduce the array output according to its efficiency. For example, if it is only 95% efficient, reduce the output to 1.54 kWh/W_p. If a maximum power point tracker is not used, then the useful array output would be diminished by around 20 to 25%, to, say, 1.21 kWh/W_p.
- A further reduction of a few percent for wiring losses may be necessary—assume these to be 3% in this example, so that useful output is 1.49 kWh/W_p.
- The portion of the electricity that will transit the battery en route to the load must be reduced by the battery inefficiencies. A round-trip energy efficiency of 80% is reasonable for PV charging in a hybrid system. For this example, assume that it is expected that half the PV output will pass through the battery and the other half go directly to the load. Then 50% of the available array output will suffer additional 20% losses, bringing the total useful array output to 1.34 kWh/W_p (i.e., [80% · 50% + 50%] · 1.49 kWh/W_p).
- That portion of the array electricity going directly to the load that must pass through an inverter should be adjusted according to the inverter efficiency. Assume for this example that 50% of the array output goes to the load directly, that 30% of the load is AC, and that the inverter is roughly 90% efficient. Then the useful array output is 1.32 kWh/W_p (i.e., (80% · 50% + (90% · 30% + 70%) · 50%) · 1.49 kWh/W_p).

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[2] It is also possible to account for any salvage value associated with the array at the end of the project by converting the salvage value to its present worth according to the discount rate and subtracting this from $c_{array\ capacity}$.

[3] If the low light performance of the array is known to be poor, the array output should be reduced further, especially in areas characterized by overcast skies.

Finally, multiply by the duration of the project to determine the useful production of a unit of array capacity over the course of the project. In the above example, if the system would be used for 20 years, then E_{array} would be 20 · 1.32 kWh/W_p, or 26.4 kWh/W_p. If the installed cost of the array was \$8/W_p, then the per unit cost of generating electricity with the PV array assuming no waste, C_{PV}, would be \$8/Wp divided by 26.4 kWh/Wp, or \$0.30/kWh.

DETERMINING THE COST OF GENERATING ELECTRICITY WITH THE GENSET

The per unit cost of providing power to the load with the genset, C_{genset}, must be directly comparable to C_{PV}. This creates a complication: while the array is paid for at the beginning of the project, the fuel and maintenance expenses for the genset occur over the project lifetime. This stream of future costs must be converted to the present. To do this, the present value of the fuel and maintenance costs associated with generating 1 kWh in each year of the project is calculated and divided by the number of years in the project:

$$C_{genset} = \frac{PV\left(r_{fuel}, n, c_{fuel}\right) + PV\left(r_{maintenance}, n, c_{maintenance}\right)}{n}$$

where $PV(\ )$ denotes the function for finding the present value of a stream of future costs, c_{fuel} is the fuel cost per kWh of genset output at the outset of the project (i.e., not accounting for the time value of money or inflation), $c_{maintenance}$ is the maintenance cost per kWh of genset output at the outset of the project, r_{fuel} and $r_{maintenance}$ are the discount rates to be used for fuel and maintenance expenditures, respectively, and n is the project lifetime, in years.

The per unit output cost for fuel, c_{fuel}, can be calculated in the following manner:

- Determine the specific fuel consumption for the genset at its typical operating point. This may be found directly or indirectly from the genset specifications. For example, consider a 10 kW diesel genset that consumes 6 litres per hour of full load operation and 1 litre per hour of idling; based on the system configuration and loads, it is expected to operate at around 60% load factor most of the time that it is on. Were no other information available, a good estimate of its fuel consumption in an hour of operation would be found by assuming that fuel consumption varies linearly between the two points known: at 60% part load, fuel consumption would be 1 l for idling plus 60% · (6 l – 1 l) for providing power, or 4 l per hour. In that hour the genset would generate 60% · 10 kWh, or 6 kWh. So the specific fuel consumption would be 4 l / 6 kWh, or 0.67 l/kWh.
- Adjust the specific fuel consumption for the battery efficiency and battery charger efficiency. For example, imagine that the above genset will be used in a system where it is expected that 90% of the genset output will go into battery charging and the remaining 10% will go to loads that are 30% AC and 70% DC; assume a round-trip battery efficiency of 80% and a charger efficiency of 90%. Then the fuel consumption per useful kWh of output is 0.67 l/kWh / (90% · 80% + 10% · (30% + 70% · 80%)) or 0.67 / 81% = 0.83 l / kWh.
- Multiply the specific fuel consumption by the delivered cost of fuel. If diesel fuel cost \$1.20/l in the above example, then the per unit output cost for fuel is \$0.99 / kWh.

The per unit output cost for maintenance, $c_{maintenance}$, can be calculated as follows:

- Determine the typical number of hours that the genset will operate to produce a kWh of useful output. In the above example, the 10 kW genset will typically run at a load factor of 60%. Thus, in one hour it generates 6 kWh. But this must be reduced by the inefficiencies in the battery and the charger: 6 kWh · 81 % yields useful output of 4.8 kWh each hour. The reciprocal of this indicates that the genset operates 0.2 hours for each kWh generated.
- Determine the cost of genset maintenance per hour operated. For example, routine maintenance may cost around \$1 per hour of operation including the technician's travel time. In addition, the diesel genset may need to be overhauled every, say, 10,000 hours of operation; assume that the

overhaul costs \$2000 all told. Then maintenance costs would be \$1 per hour + \$2000/10,000 hours, or \$1.20 per hour[4].

- Then the per unit output cost for maintenance is the product of the two, e.g., 0.2 h/kWh · \$1.20/h yields \$0.25/kWh.

Now all that remains in the calculation of C_{genset} is the application of the present value function (e.g., as found in most spreadsheet software) for a stream of yearly costs. The only difficulty is accounting for inflation, for the maintenance costs, and the fuel cost escalation rate, for fuel costs. To be rigorous, the stream of annual fuel and maintenance costs should be inflated, year-on-year, to account for these. In reality, little error will result if the discount rate used in the present value function is simply the difference between the project discount rate and the appropriate inflation rate. Thus, r_{fuel} is the project discount rate minus the expected inflation rate for fuel expenditures, and $r_{maintenance}$ is the project discount rate minus the expected inflation rate for parts and labour.

To continue with the above example, if a project life of 20 years, an inflation rate of 2%, a fuel cost escalation rate of 3% and a discount rate of 10% is assumed, then the per unit cost of providing power to the load with the genset would be:

$$\begin{aligned} C_{genset} &= \frac{PV(10\%-3\%, 20, 0.99) + PV(10\%-2\%, 20, 0.25)}{20} \\ &= \frac{10.49 + 2.45}{20} \\ &= \$0.65\,/\text{kWh} \end{aligned}$$

INTERPRETATION OF $f_{PVWaste}$

Now the cost to generate a unit of output with the genset and with the PV array (assuming nothing rejected by the charge controller) is known. The ratio of the two is surprisingly informative: if C_{genset}/C_{PV} is less than one, then a hybrid system is not strictly cost-effective compared with a cycle charging system. The genset can generate electricity more cheaply than can the array, even when none of the array's output is rejected.

If the ratio is greater than one, then some amount of PV array capacity makes sense. But how much? We can use the ratio to calculate $f_{PVWaste}$, as shown above. This marginal waste factor reveals exactly how large to make the array: we want to add array capacity until the wasted portion of the output of the last unit added is equal to $f_{PVWaste}$. Beyond that point, it is cheaper to generate electricity with the genset than the array.

In practice, how can the wasted fraction of the output of the last unit of PV array capacity be determined? Sizing or simulation software can help; indeed, knowing $f_{PVWaste}$ is helpful in using these programs and could even guide their optimization routines. Using software is not necessary, however, at least for sites where there is significant monthly variation in either the load or the available solar energy.

SIZING THE ARRAY USING MONTHLY RADIATION DATA

To see how knowledge of $f_{PVWaste}$ can be used to size the array when only the monthly plane-of-array solar radiation is known, consider the above example from Winnipeg, powering a load of 4.8 kWh per day throughout the year. The average daily radiation in the plane of the array is shown, month-by-month, in Table 1. For each month, this is expressed as a fraction of the annual total; for instance, 10.4% of the year's solar radiation strikes the array during the month of July, the sunniest month of the year.

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[4] Strictly speaking, the present value of the future overhaul cost should be used. Unfortunately, the sizing of the array affects how soon the genset needs overhauling. A rough estimate of the overhaul interval, based on past experience, can be used. Typically overhaul is a very minor part of genset costs.

	Average Daily Solar Radiation in Plane of Arr. (kWh/m²/day)	Fraction of annual solar radiation	Fraction of annual output wasted by last unit of PV	Critical Array Size (W$_p$)	
January	3.59	6.55%		1692	
February	4.83	8.81%		1257	
March	5.66	10.31%	10.31%	1074	
April	5.45	9.94%	9.94%	1114	
May	5.39	9.84%	9.84%	1126	
June	5.39	9.82%	9.82%	1128	
July	5.69	10.38%	10.38%	1067	
August	5.38	9.81%		**1129**	Optimal
September	4.42	8.05%		1375	
October	3.46	6.31%		1754	
November	2.81	5.12%		2163	
December	2.78	5.06%		2186	
	Total fraction of annual output wasted by last unit of PV		50.29%		

Table 1. Sizing of the Array for a 4.8 kWh/day Load in Winnipeg

At a certain critical array size, the array output on an average day will be just sufficient to satisfy the daily load and all associated losses (in storage, conversion, etc.). As a first approximation, assume that when the array is smaller than the critical size for a given month, none of the array's output is rejected by the charge controller during the month; conversely, assume that when the array is larger than the critical size, all output of that portion of the array larger than the critical size is wasted. In reality, this is not the case: some waste occurs well before the array reaches this critical size, and some of the additional output of an array larger than the critical size goes towards satisfying the load. But the errors in one month tend to compensate for the errors in another, such that this is a good approximation.

Then, according to this approximation, if the array is at the critical size for the month of July, and one unit of PV array capacity is added, then the entire output of this additional unit is wasted for the month of July. In the other months, however, its entire output will be used. Thus, the fraction of its output that is wasted is simply the portion of the solar radiation that strikes the array in the month of July, or 10.4% (assuming that the array output is directly proportional to the average daily radiation—a reasonable approximation). If the array is sized just slightly larger than the critical size for March, the second sunniest month, then the fraction wasted will be 20.7%, and so on.

For our example, C_{PV} is \$0.30/kWh and C_{genset} is \$0.65/kWh; thus, $f_{PVWaste}$ is 0.54. That is, we want to add PV array capacity until the last unit added has 54% of its output rejected by the charge controller. Examining the table of monthly radiation values, we see that the period of March through July (the five sunniest months of the year) accounts for 50.3% of the year's solar radiation. This matches our target quite well. If the array is sized for August, the sixth sunniest month, then the fraction of the additional array capacity's output that is wasted will be around 50 to 55%. This is, therefore, the financially optimal array sizing.

To translate this target into an actual array sizing, use the ratio of useful array output to solar radiation, based on the calculations for E_{array}. The critical array size in a given month is, therefore:

$$s_{array} = \frac{l_{day} \cdot e_{solar}}{e_{useful} \cdot e_{month}}$$

where s_{array} is the array capacity, in W$_p$; l_{day} is the average daily load for the month, in kWh/day; e_{solar} is the annual solar radiation in the plane of the array, in kWh·m^{-2}·year^{-1}; e_{useful} is the annual useful output of a

1 W_p array, accounting for all inefficiencies and losses but assuming nothing is rejected by the charge controller, in kWh·W_p·year^{-1}; and e_{month} is the average daily radiation for the month, in kWh·m^{-2}·day^{-1}.

For instance, the critical array size for the month of August from our example is:

$$s_{array} = \frac{4.8\ \text{kWh}\cdot\text{day}^{-1}\cdot 1670\ \text{kWh}\cdot\text{m}^{-2}\cdot\text{year}^{-1}}{1.32\ \text{kWh}\cdot\text{W}_p^{-2}\cdot\text{year}^{-1}\cdot 5.38\ \text{kWh}\cdot\text{m}^{-2}\cdot\text{day}^{-1}}$$
$$= 1130\ \text{W}_p$$

Therefore, the optimal array sizing for this example is 1130 W_p.

The use of monthly data can be extended to work for situations where the load varies monthly or seasonally. Imagine that the load for the Winnipeg example above varied monthly as shown in Table 2. For each month, the critical array size can be calculated according to the above formula. Then, the months can be ranked, from smallest critical array size to largest. In Table 2, the load is so low in December that it has the smallest critical array size, despite being the month with the least solar radiation; July has the fourth largest critical array size. If $f_{PVWaste}$ is 0.54, as before, we would find the m months with the highest rankings that together account for roughly 54% of the solar radiation. It turns out for this example that m is six. The sixth month is February, with a critical array size of 1126 W_p, and the seventh is September, at 1146 W_p. The optimal array sizing lies between the two; it is purely coincidental that this sizing coincides with the sizing for a fixed daily load of 4.8 kWh/day.

	Average Daily Solar Radiation in Plane of Arr. (kWh/m^2/day)	Average daily load (kWh/day)	Critical Array Size (W_p)	Rank in terms of critical array size (1=smallest)	Fraction of annual solar radiation	Fraction of annual output wasted by last unit of PV	
January	3.59	3.0	1057	5	6.55%	6.55%	
February	4.83	4.3	**1126**	6	8.81%	8.81%	
March	5.66	4.7	1051	4	10.31%	10.31%	
April	5.45	5.5	1276	10	9.94%		Optimal size between size for February and September
May	5.39	5.2	1220	8	9.84%		
June	5.39	4.3	1010	3	9.82%	9.82%	
July	5.69	5.5	1222	9	10.38%		
August	5.38	4.0	941	2	9.81%	9.81%	
September	4.42	4.0	**1146**	7	8.05%		
October	3.46	4.8	1754	11	6.31%		
November	2.81	4.7	2118	12	5.12%		
December	2.78	2.0	911	1	5.06%	5.06%	
				Total fraction of annual output wasted by last unit of PV		50.36%	

Table 2. Sizing of the Array for a Load that Varies by Month

FURTHER INTPRETATION OF $f_{PVWaste}$

Given all the approximations that were made in arriving at this optimal array sizing, should we be concerned about the accuracy of the estimate? Or what if, for aesthetic reasons, an array of 1050 or 1200 W_p would be better—would this be a major concern?

These questions can be addressed by returning to the informative ratio of C_{genset}/C_{PV} introduced earlier. When this ratio is near one, it indicates that the cost of generating with PV is very similar to that of generating with the genset. So using a larger array size than is optimal is likely to be costly; using a smaller array is not associated with a significant penalty. But in the Winnipeg example, the ratio is 0.65/0.3, or greater than two. In this case, there is less latitude for using a smaller array, but increasing the size of the array, perhaps by 15 or 20% over the optimal sizing, may be acceptable.

Whether or not a larger array incurs a significant cost penalty depends not just on the ratio of C_{genset}/C_{PV}, but also on how suddenly the curve for genset costs levels out with increasing array size. This can be estimated by examining the range of critical array sizes: if it is large[5], then the curve levels out gradually, since larger arrays can make a contribution in those months with larger critical arrays. In the Winnipeg examples, the curve levels out gradually—with the fixed load, the critical array size for December is twice that of July, and with the varying load, the range is even wider. But at a more equatorial site, where the month-to-month variation in the solar radiation is minimal, the curve would level off more abruptly, and using a larger than optimal array could significantly increase overall costs. This situation can also arise at mid- and high-latitude sites where the monthly load is strongly correlated with the available solar radiation.

The ratio of C_{genset}/C_{PV} also provides a hint, in some cases, as to whether a PV-battery system should be contemplated in place of a hybrid system. When $f_{PVWaste}$ approaches unity, using photovoltaics is far more cost-effective than using the genset. But the reverse is not always true: just because $f_{PVWaste}$ is low does not mean that a PV-battery system is out of the question. If the load is small, then eliminating the genset may free up enough capital to invest in the large array and battery necessary for a PV-battery system. Since the capital costs of the genset do not figure in C_{genset}, this is not captured by this ratio.

ACCOUNTING FOR BATTERY WEAR

When the fraction of the array output that is stored in the battery differs greatly from the fraction of the genset output that goes towards charging, the cost of battery wear may slightly affect the optimal array sizing. The above method can accommodate this, but it requires estimates of the fractions of the array and genset output destined for the battery. In some cases, this may require simulation.

While generally unnecessary, an accounting of battery wear can be included in the above method:

- First, ensure that the cycle life, and not the calendar life, limits the battery lifetime.
- Calculate a cost for battery usage based on the manufacturer's specifications for cycle lifetime and the cost of the battery. For example, if a 16 kWh battery will be cycled to around 60% depth-of-discharge, and is purported to achieve 500 cycles at this depth, then the total energy that can be liberated from the battery over its lifetime is 500 cycles · 16 kWh· 60%, or 4800 kWh. Since we have assumed a battery efficiency of 80%, this 4800 kWh liberated from the battery corresponds to 6000 kWh fed into the battery. If the battery costs \$3000 installed, then the cost for wear, c_{wear}, is \$0.50 per kWh of genset or PV output stored in the battery.
- As for C_{genset}, calculate a present value for battery wear based on the project lifetime:

$$C_{battery} = \frac{PV\left(r_{maintenance}, n, c_{wear}\right)}{n}$$

  In the Winnipeg example, based on c_{wear} of \$0.50 per kWh, $C_{battery}$ is \$0.25 per kWh.
- Now increase C_{PV} and C_{genset} according to $C_{battery}$ and the fraction of each kWh of output that is destined for the battery. In the Winnipeg example, 50% of the array output goes to the battery, so C_{PV} should be increased by \$0.13 per kWh (to \$0.43/kWh), and 90% of the genset output goes to the battery, so C_{genset} should be increased by \$0.22 per kWh (to \$0.87 per kWh)[6].
- Recalculate $f_{PVWaste}$ and resize the array. For the Winnipeg example, $f_{PVWaste}$ becomes 0.51 instead of 0.54. This would have no effect on array sizing.

ADJUSTING THE ARRAY SIZING

The above method finds the financially optimal array size, at least insofar as all the relevant costs of the array and genset have been included for consideration. It has been impossible to include certain

[5] Actually, not only must the range be large, but also the critical array sizes must fall over the entire range, and not just be clustered at one point with one or two outliers.

[6] Not having taken into account adjustments associated with the battery charger efficiency and the inverter efficiency, these are not rigorous estimates, but they are sufficiently close.

important costs related to battery longevity, however, so it is likely that, wherever permitted by the ratio of C_{genset}/C_{PV} and the abruptness with which the genset fuel costs curve levels off, the array size should be increased by 10 to 25% beyond that implied by $f_{PVWaste}$. A system with a larger array will enjoy operational benefits, be better for the environment, and be more insulated from risks associated with fuel cost escalation or load increases.

The lead-acid batteries typically used in PV-hybrid systems often have curtailed lifetimes when cycled repeatedly between partial states-of-charge, without regular full charging. Ideally, they would be fully recharged on every cycle. To do this with the genset is very expensive, however, since as the battery approaches a full state-of-charge, its ability to accept charge decreases: the genset must operate for four to eight hours at potentially very low loading levels, with associated fuel consumption and heightened wear. Using a larger array increases the likelihood of full charging by PV during the months when the critical array size is low, such that the genset may not need to operate at low part load during this time. It also improves the chances of the occasional full charge by PV in the other months.

It is difficult to account for the effects of partial state-of-charge cycling on longevity even in simulation software, let alone in a simplified method. These costs are real, however, and can be very significant. It makes sense, therefore, to use an array size larger than implied by $f_{PVWaste}$ wherever possible.

CONCLUSIONS

The method presented here reveals three useful pieces of information:

1) It indicates whether a PV-hybrid system is more cost-effective than a genset-battery system.
2) It identifies the most cost-effective array sizing.
3) It gives an indication of how critical this sizing is: that is, whether there is latitude to increase the array size with negligible impact on overall costs.

With the availability of free software for sizing and simulation, why bother with this method? For one, software programs are not gods, and they sometimes hide errors that may be hard to detect. For instance, based on its rules of thumb, one very popular software tool suggests that the array for the Winnipeg example should be 550 W_p—less than half as large as it should be! The error is significant: changing from the 550 W_p array to the 1130 W_p array decreased the present value of costs for array, genset, fuel and maintenance by over $5000, as calculated by the software package itself.

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