Optimal Wire Size for Photovoltaic Systems Operating at Maximum Power Point: A Closed Form Approach

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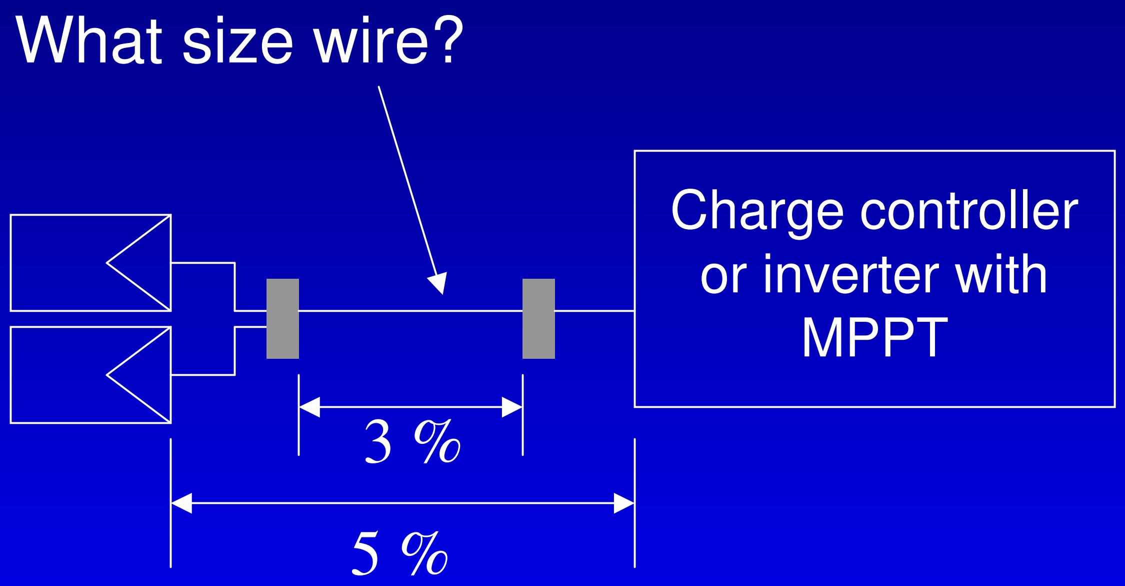
Wire size selection by voltage drop rule



(& check ampacity)

Optimal wire size: Why bother?

- Wiring costs are generally, BUT NOT ALWAYS, minor part of PV system installation cost

- Questions like:
  - "If array is far away, should I minimize voltage drop to 1% or let it be the maximum 3 or 5%?"
  - "Is it worthwhile to raise the system voltage to reduce wiring losses and wiring costs?"
  - "I just paid 3 squigzillion dollars for my PV system, don't I want to minimize the losses in the wiring to the absolute minimum?"

- Because a simple, more rigorous alternative exists!

Two independent parts of approach

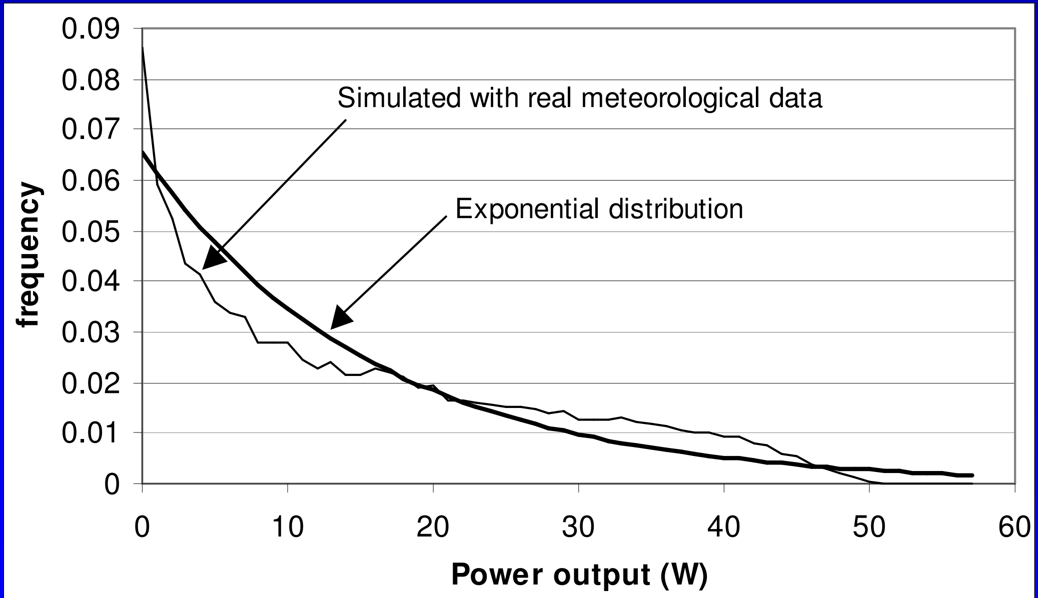
1. Use of a simple distribution to estimate average array output
2. Formulation of optimization problem

Instantaneous and average power losses

- Instantaneous power loss in cable:

$$P_{loss} = \frac{P_{array}^2 R_{wire}}{V_{array}^2}$$

- But average power loss is not necessarily found with average array power
  - Need to know frequency distribution of array output



Assumed frequency distribution of array power

$$f\left(p_{array}\right) = Ce^{\gamma p_{array}}$$

- Bendt et al., 1981, for clearness index
- Peippo and Lund, 1994, applied it to power output of array
- Constants found from average and maximum output of array
- Not too accurate, but seems to work surprisingly well
- IT CAN BE INTEGRATED!

Calculation of f_{site}, "cabling loss factor"

$$\overline{P}_{loss} = \int_0^{p_{\max}} \frac{\left(\hat{P} \cdot p_{array}\right)^2 R_{wire} f\left(p_{array}\right)}{V_{array}^2} dp_{array}$$

$$= \frac{CR_{wire}\hat{P}^2}{V_{array}^2} \int_0^{p_{\max}} p_{array}^2 e^{\gamma p_{array}} dp_{array}$$

$$= \frac{CR_{wire}\hat{P}^2}{V_{array}^2} \left[\frac{e^{\gamma p_{\max}}}{\gamma} \left(p_{\max}^2 - \frac{2p_{\max}}{\gamma} + \frac{2}{\gamma^2} \right) - \frac{2}{\gamma^3} \right]$$

$$= \frac{f_{site} R_{wire} \hat{P}^2}{V_{array}^2} \quad where \ \ f_{site} = C\left[\frac{e^{\gamma p_{\max}}}{\gamma} \left(p_{\max}^2 - \frac{2p_{\max}}{\gamma} + \frac{2}{\gamma^2} \right) - \frac{2}{\gamma^3} \right]$$

Interpretation of cabling loss factor, f_{site}

- Magnitude of average cabling power losses as a fraction of output of array were it always to put out its rated power
- Example:
  - Array puts out 1000 Wh/Wp per year
  - 4000 hours of daylight per year
  - Maximum array output is its rated power
  - Then cabling loss factor is 0.11
  - Average losses in cable only 11% of losses at rated output

Validation through PVToolbox simulation

- CWEEDS meteorological data for Vancouver, Inuvik, Edmonton, Toronto, St. John’s
- Watgen synthesized data for Albuquerque, Athens, El Fasher (Sudan)
- Horizontal, south-facing latitude tilt & vertical, west-facing vertical, east-facing 45º
- Average error of –7.1%

Formulation of optimization problem

- Total wiring costs = cost of cable + cost of loss
- How do we express the cost of future stream of losses as a present value?
  - Use cost of the PV capacity that generates this amount of power, on average

$$
\begin{aligned}
c_{tot} &= c_{loss} + c_{wire} \\
&= \frac{\overline{p}_{loss}}{\overline{p}_{array}} \hat{P} \cdot c_{pv} + c_{wire}\left(R_{wire}\right) \\
&= \frac{f_{site} R_{wire} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + c_{wire}\left(R_{wire}\right)
\end{aligned}
$$

Formulation of optimization problem, cont.

- We can express this on a per unit length of wire basis:

$$c_{tot} = \frac{f_{site} l_{wire} r_{wire} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + c_{wire}(r_{wire}) \cdot l_{wire}$$

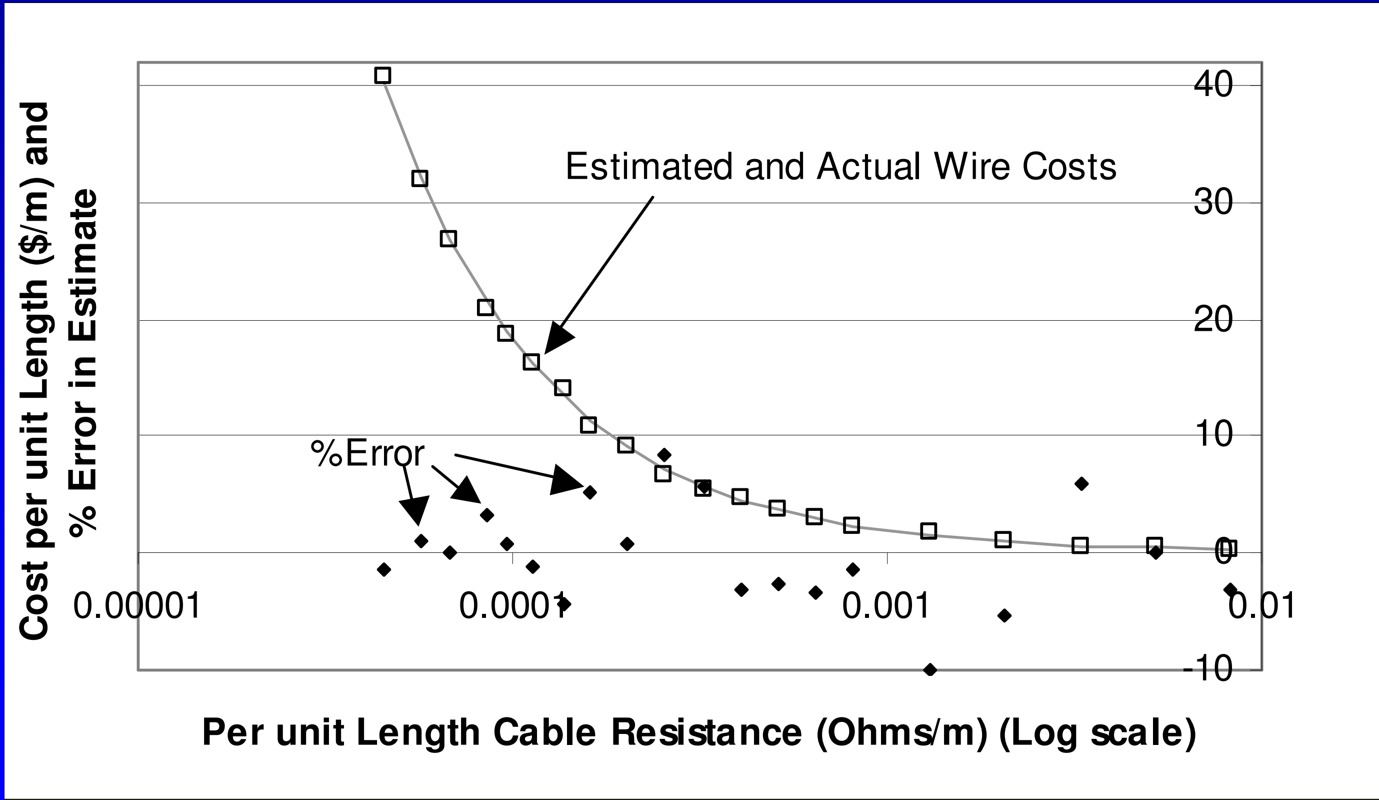
- Set derivative to zero to minimize:

$$\frac{dc_{tot}}{dr_{wire}} = \frac{f_{site} \hat{P}^2}{V_{array}^2 \overline{p}_{array}} c_{pv} + \frac{dc_{wire}}{dr_{wire}} = 0$$

- We can divide length of the wire out of equation!

Cost of cable

- Assume: $$c_{wire}(r_{wire}) = c_{wfix} + \frac{c_{wvar}}{r_{wire}}$$



Optimal cable resistance

$$\frac{f_{site}\hat{P}^2}{V_{array}^2\,\overline{p}_{array}}c_{pv} - \frac{c_{wvar}}{\hat{r}_{wire}^2} = 0$$

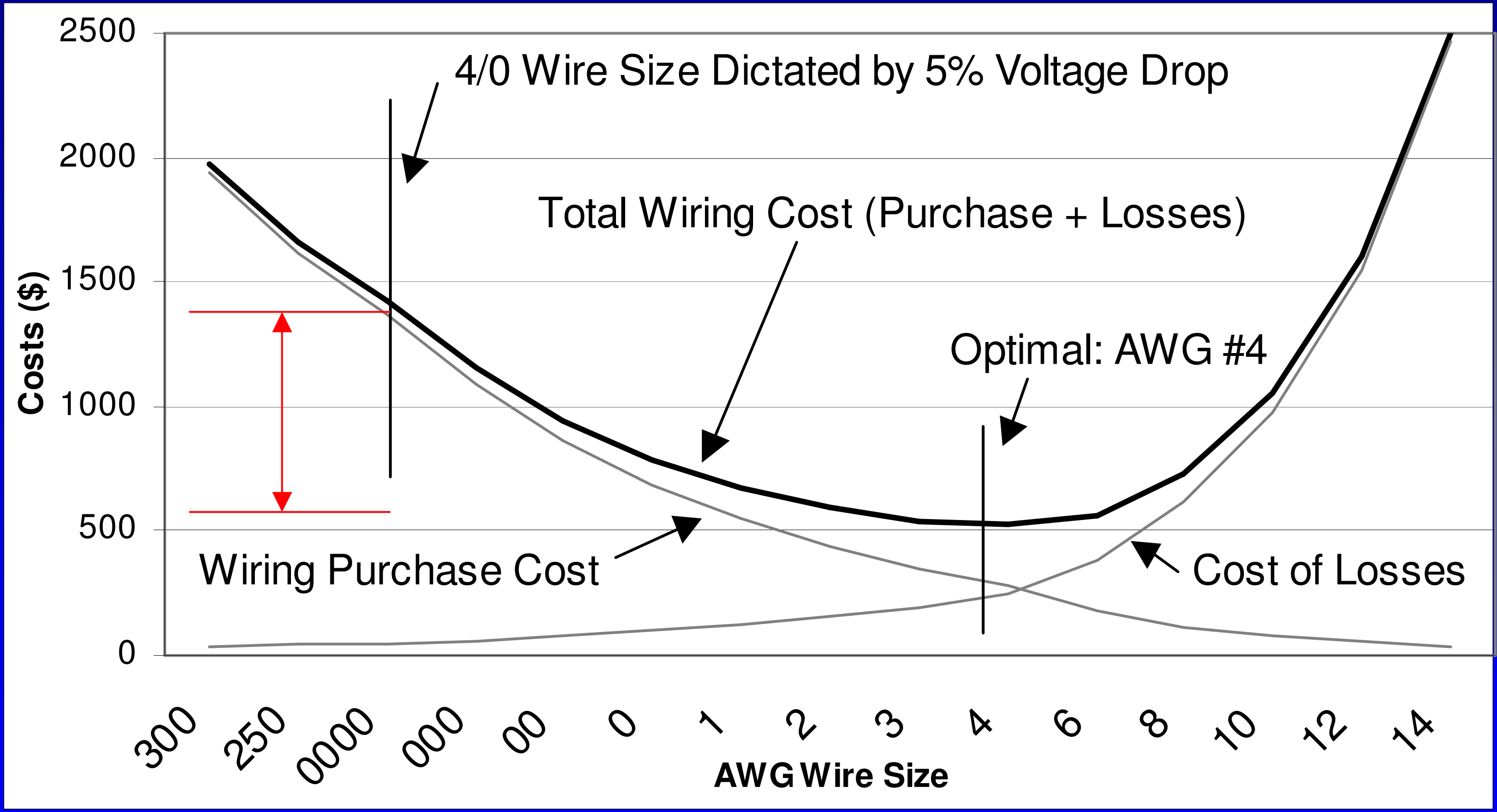
$$\hat{r}_{wire} = \frac{V_{array}}{\hat{P}}\sqrt{\frac{\overline{p}_{array}c_{wvar}}{f_{site}c_{pv}}}$$

- Example:
  - Cabling loss factor of 0.11, cost of cable as shown previously, cost of PV capacity \$10/Wp.

$$\hat{r}_{wire} = \frac{V_{array}}{49\hat{P}}$$

Example application: small PV system

- 360 Wp array at $10/Wp, 12 V, Vancouver, 60 m to array



Difference is 20 % of system cost !

Example application: raise voltage to 48 V

- Previous example, but shift array voltage to 48 V using a charge controller that permits this
  - Manufacturer claims $1000 cost of cable savings using 5% rule

$$\hat{r}_{wire} = \frac{V_{array}}{53.5 \cdot \hat{P}}$$

- Optimal wire size shifts from AWG #4 at 12 V to AWG #10 at 48 V
  - Cost of cable savings: $200
  - Total wiring costs savings: $400

Implications for Canadian Electrical Code

- Code enforces 3% and 5% voltage drop rule
  - Implies that length of cable is a consideration
- Rationale: voltage drop overheats motors in AC systems
  - Rationale does not apply to PV systems
- Code is a safety standard, not a performance standard
  - Make an exemption for PV systems with maximum power point tracking

Conclusions

- A simple relation exists for optimal wire size, and it is useful for discussing issues related to system voltage
- Optimal cable size is independent of wire length
- In some systems, use of voltage drop to size wire can add 20% to the cost of the system
- Canadian Electrical Code should exempt PV system with MPPT from voltage drop rule